(Following Paper ID and Roll No. to be filled in your Answer Books)

Paper ID : 199222 Roll No.

B. TECH.

Theory Examination (Semester-II) 2015-16

ENGG MATHEMATICS-II

Time : 3 Hours *Max. Marks : 100*

Section-A

Note: Attempt all questions of this section. (2×10 = 20)

1. (a) Find the roots of the auxiliary equation of the differential equation.

$$\frac{d^2y}{dt^2}-6\frac{dy}{dt}+9y=4e^{3t}$$

(b) Find the order and degree of the following differential equation

$$\frac{d^2y}{dx^2}+\sqrt{1+\left(\frac{dy}{dx}\right)^3}=0$$

Also explain your answer.

(c) Find the values of m and n, if

$$3x^2 = mP_2(x)+nP_0(x)$$

(d) Write the statement of Rodrigue formula for Legendre function.

(e) Find Inverse Laplace Transform of the function

$$f(s)=\frac{s}{2s^2+8}$$

(f) Find the Laplace transform of unit step function u(t-a)

(g) Find the value of the Fourier coefficient a_0 for the function

$$f(x)=\begin{cases}0, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$$

(h) Find the particular integral of the following partial differential equation

$$(D^2+DD'-6D'^2)z=\cos(2x+y)$$

(i) Write two-dimensional heat equation.

(j) Classify the following partial differential equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Also explain your answer.

Section-B

2. Attempt any five questions from this section.

(10×5 = 50)

(a) Solve the following simultaneous equations

$$\frac{d^2x}{dt^2}+y=\sin t$$

$$\frac{d^2y}{dx^2}+x=\cos t$$

(b) (i) Using variation of parameter method, solve

$$x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-12y=0$$

(ii) Obtain the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^3e^x$$

(c) Find the series solution of the following differential equation.

$$2x(1-x)\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+3y=0$$

(d) State convolution theorem of Laplace transform and using it find :

$$L^{-1}\left\{\frac{1}{(s^2+4)(s+2)}\right\}$$

(e) Solve the Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

in a rectangle in the xy-plane, $0\le x\le a$ and $0\le y\le b$ satisfying the following boundary conditions

$u(x,0)=0, u(x,b)=0$

$u(0,y)=0$ and $u(a,y)=f(y)$

(f) Find the fourier series to represent the function $f(x)$ given by

$$f(x)=\begin{cases}-k \text{ for } -\Pi<x<0\\ k \text{ for } 0<x<\Pi\end{cases}$$

Hence show that

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\dots\dots\dots=\frac{\Pi}{4}$$

(g) Show that

$$J_n(x)=\frac{1}{\Pi}\int_0^{\pi}\cos(n\varphi-x\sin\varphi)\,d\varphi),$$

n being positive integer and $J_n(x)$ is Bessel function.

(h) Solve the following partial differential equation :

$$(D^2-DD^{I}-2D^{II}+2D+2D^{I})z=Sin(2x+y)$$

where notations have their usual meaning.

Section-C

Note: Attempt any two questions from this section.

(15×2=30)

3. (a) Solve $(D^2-2D+1)y=e^x\sin x$

(b) Show that $(1-2xz+z^2)^{-1/2}=\sum_{n=0}^{\infty}z^n P_n(x)$

(c) Prove that

$$\frac{d}{dx}\left[x^nJ_n(x)\right]=x^nJ_{n-1}(x)$$

4. (a) Apply Laplace transform to solve the equation

$$\frac{d^2y}{dt^2}+y=t\cos 2t,\, t>0$$

given that $y=\frac{dy}{dt}=0$ for t=0

(b) Find the Laplace transform of

(i) $L\{t^2\}$

(ii) L {cosh at.cos bt}

(c) Solve the following differential equation

$$(D^3 - 1)y = 3x^4 - 2x^3$$

5. (a) Solve the following partial differential equation

$$(y^2 + z^2)p - xyq + zx = 0$$

where p and q have their usual meaning.

(b) Find the Fourier series of

$$f(x) = x^3 \text{ in } (-\Pi, \Pi)$$

(c) Classify the following partial differential equation

$$(1 - x^2)\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + (1 - y^2)\frac{\partial^2 z}{\partial y^2} - 2z = 0$$