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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9959 Roll No.

B. Tech.

(SEM. III) EXAMINATION, 2008-09

MATHEMATICS - III

Time : 3 Hours] *[Total Marks : 100*

Note : *(1) Attempt all questions.*
(2) Marks are shown against each question.

1 Attempt any **four** of the following : 5×4=20

(a) Find the Fourier sine integral for

$$f(x) = e^{-\alpha x} \qquad (\alpha > 0)$$

Hence show that

$$\frac{\pi}{2} e^{-\alpha x} = \int_0^\infty \frac{\lambda \sin(\lambda x)}{(\alpha^2 + \lambda^2)} d\lambda$$

(b) Find the Fourier transform of

$$f(x) = \begin{cases} x, & |x| \le a \\ 0, & |x| > a \end{cases}$$

(c) Find the Fourier sine and cosine transforms of

$$f(x) = x, \text{ for } 0 < x < \frac{l}{2}$$

$$= 1 - x \text{ for } \frac{l}{2} < x < l$$

$$= 0 \text{ for } x > l.$$

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(d) Use Fourier transform to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0, t) = 0$ and

$$u(x, 0) = \begin{cases} 1, & 0 < x < 1, \\ 0, & x \geq 1 \end{cases}$$

(e) Find the Z-transform of $\sin(\alpha k)$, $k \geq 0$.

(f) Solve by Z-transform :

$$y_{k+1} + \frac{1}{4} y_k = \left(\frac{1}{4}\right)^k, \ (k \geq 0), y(0) = 0$$

2 Attempt any **four** of the following : 5×4=20

(a) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function, show that the family of curves $u = c_1$ and $v = c_2$ intersect orthogonally.

(b) Find an analytic function whose imaginary part is $e^{-x}(x \cos y + y \sin y)$

(c) If $f(z)$ is a harmonic function of z, show that

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2$$

(d) Evaluate $\int_0^{2+i} (\bar{z})^2 \, dz$, along

(i) the real axis to 2 and then from 2 to $2 + i$

(ii) the line $y = x/2$

(e) Use Cauchy's integral formula to evaluate

$$\int_C \frac{z}{(z^2-3z+2)}dz$$

where C is the circle $|z-2|=\frac{1}{2}$.

(f) State and prove Liouvilles Theorem.

3 Attempt any **two** of the following : 10×2=20

(a) Evaluate $\int_C \frac{z^2-2z}{(z+1)^2(z^2+4)}dz$

where C is the circle $|z|=10$.

(b) Using residue theorem, show that

$$\int_0^{2\pi} \frac{d\theta}{a+b\sin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$$

where $a>|b|$

(c) Consider the transformation $w=z^2$.

(i) Find the image of the first quadrant in the z-plane under this transformation.

(ii) Show that the straight lines $x=$ const. in the z-plane are mapped into a family of parabolas in the w-plane

4 Attempt any **two** of the following : **10×2=20**

(a) If the coefficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}(3/8)$, show that $\sigma_x = \frac{1}{2}\sigma_y$

(b) In a certain factory manufacturing razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use suitable distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets.

(c) Prove that for normal distribution the mean deviation from the mean equals to 4/5 of the standard deviation.

5 Attempt any **two** of the following : **10×2=20**

(a) Solve $x^4 + 12x - 5 = 0$

(b) Using the method of least squares, fit a straight line from the following data :

x	0	2	4	5	6
y	5.012	10	15	21	30

(c) Fit a parabola of the form $y = a + bx + cx^2$ to the data :

x	1	2	3	4
y	1.7	1.8	2.3	3.2

by the method of least squares.