

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199403

Roll No.

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B.Tech.

(SEM. IV) THEORY EXAMINATION 2013-14

MATHEMATICS—III

Time : 3 Hours

Total Marks : 100

Note :— Attempt all questions.

1. Attempt any four parts of the following : (4×5=20)

- (a) Show that $f(z) = \log z$ is analytic everywhere in the complex plane except at the origin.
- (b) Find an analytic function whose imaginary part is $e^{-x}(x \cos y + y \sin y)$.
- (c) Use Cauchy's integral formula to show that

$$\int_c \frac{e^{zt}}{z^2 + 1} dz = 2\pi i \sin t \text{ if } t > 0 \text{ and } c \text{ is the circle } |z| = 3.$$

- (d) Obtain the Taylor's series expansion of $f(z) = \frac{1}{z^2 - 4z + 3}$ about the point $z = 4$. Find its region of convergence.

- (e) Determine the poles and residues at each pole for

$$f(z) = \frac{z-1}{(z+1)^2(z-2)} \text{ and hence evaluate } \oint_c f(z) dz \text{ where } c \text{ is the circle } |z - i| = 2.$$

(f) Evaluate :

$$\int_0^{2\pi} \frac{\cos 3\theta}{5 + 4 \cos \theta} d\theta.$$

2. Attempt any two parts of the following : $(2 \times 10 = 20)$

(a) The following table represents the height of batch of 100 students. Calculate Kurtosis :

Height (in cm)	59	61	63	65	67	69	71	73	75
No. of Students	0	2	6	20	40	20	8	2	2

(b) Fit the curve $pv^r = K$ to the following data :

p (kg/cm ²)	0.5	1	1.5	2	2.5	3
v (litres)	1620	1000	750	620	520	460

(c) Ten students got the following percentage of marks in principles of Economics and Statistics :

Roll Nos.	1	2	3	4	5	6	7	8	9	10
Marks in Economics	78	36	98	25	75	82	90	62	65	39
Marks in Statistics	84	51	91	60	68	62	86	58	53	47

Calculate the coefficient of correlation.

3. Attempt any two parts of the following : $(2 \times 10 = 20)$

(a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of

the distribution. It is given that if $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{x^2}{2}} dx$, then

$$f(0.5) = 0.19 \text{ and } f(1.4) = 0.42.$$

(b) In a survey of 200 boys of which 75 were intelligent, 40 had educated fathers, while 85 of the unintelligent had uneducated father. Does this figure support the hypothesis that educated father's have intelligent boys. (Ask for χ^2 table)

(c) In a blade manufacturing factory, 1000 blades are examined daily. Draw the 'np' chart for the following table and examine whether the process is under control ?

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of Defective blades	9	10	12	8	7	15	10	12	10	8	7	13	14	15	16

4. Attempt any four parts of the following : $(4 \times 5 = 20)$

(a) Find the root of the equation $\log_{10} x - x + 3 = 0$ using bisection method correct to three decimal places.

(b) Find the rate of convergence for Regula-Falsi method.

(c) Prove that $\nabla = -\frac{\delta^2}{2} + \sqrt[8]{1 + \frac{\delta^2}{4}}$, where symbols have their usual meaning for finite differences.

(d) Obtain the missing term for following table :

x	1	2	3	4	5	6	7	8
f(x)	2	4	8	—	32	—	128	256

Explain, why the results differ from 16 and 64.

(e) The following table gives the population of a town during the last six censuses. Estimate the population in year 1913.

Year	1911	1921	1931	1941	1951	1961
Population (in thousands)	12	15	20	27	39	52

(f) Use Newton's dividend difference formula to find the value of $f(3)$ from the following data :

x	0	1	2	4	5	6
f(x)	1	14	15	5	6	19

5. Attempt any **two** parts of the following : (2×10=20)

- (a) Test if the following system of equations is diagonally dominant, and hence solve this system using Gauss-Seidal's method :

$$10x + 15y + 3z = 14$$

$$-30x + y + 5z = 17$$

$$x + y + 4z = 3.$$

- (b) (i) A rod is rotating in a plane. Following data gives the angle θ (radian) through which the rod has turned for various values of time t (seconds) :

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity of the rod at $t = 0.2$ sec.

- (ii) The velocities of a car which starts initially from rest (running on a straight road) at intervals of 2 minutes are given below :

time (minutes)	2	4	6	8	10	12
velocity (Km/hr)	22	30	27	18	7	0

Apply Simpson's $3/8$ rule to find the distance covered by the car.

- (c) Estimate $y(0.8)$ if $2y \frac{dy}{dx} = x^2$ and $y(0) = 2$, using Runge-Kutta method of fourth order by taking $h = 0.4$.