

Printed Pages—3 **ECS505**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2168 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM. V) ODD SEMESTER THEORY EXAMINATION 2012-13

GRAPH THEORY

Time : 2 Hours *Total Marks : 50*

Note : *(1) Attempt* ***all*** *questions.*

(2) Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following : **(4×3=12)**

   (a) What is a bipartite graph ? How can you determine whether an undirected graph is bipartite ?

   (b) Draw r-regular graph with eight vertices when (i) r = 3 (ii) r = 4 (iii) r = 5.

   (c) Define an Euler and Euler path in an undirected graph. How can it be determined whether an undirected graph has an Euler path ?

   (d) Prove that the number of vertices having odd degree in a graph is always even.

   (e) Discuss the travelling-Salesman problem.

   (f) What does it mean that for two simple graphs to be isomorphic ? Give an example of two graphs that have the same number of vertices, edges, and degrees of vertices, but that are not isomorphic.

2. Attempt any **two** parts of the following : **(2×6=12)**

   (a) Define a rooted tree and the root of such a tree. Define the parent of a vertex and a child of a vertex in a rooted tree.

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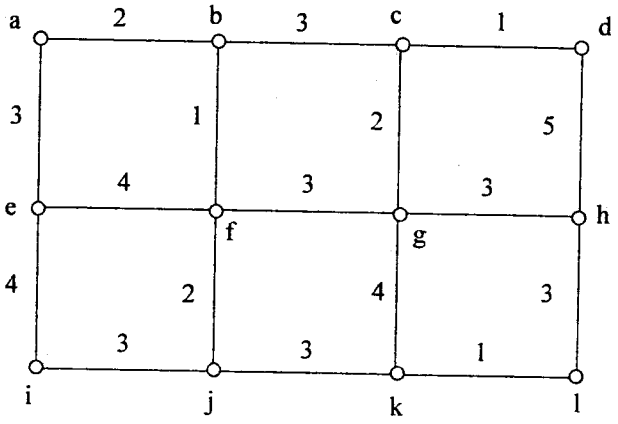
Draw a rooted tree with at least 10 vertices, where the degree of each vertex does not exceed 3. Identify the root, the parent of each vertex, and the children of each vertex.

(b) What do you mean by the rank and nullity of a graph ? Discuss the rank and nullity of a complete graph of n vertices.

(c) Use the algorithm of Prim's or Kruskal's, to find a minimum spanning tree of the following graph :



3. Attempt any **two** parts of the following : **(2×6=12)**

(a) Define the vertex connectivity and edge connectivity of a graph. Show that the maximum vertex connectivity one can achieve with a graph G of n vertices and e edges ($e \geq n - 1$) is $\lfloor 2e/n \rfloor$.

(b) What do you mean by planarity of graph G ? Discuss the Euler's formula for finding the planarity of a graph with suitable example.

(c) Define the thickness and cross number of a graph. Show, by sketching, that the thickness of the eight vertex complete graph is two, whereas that of the nine vertex complete graph is three.

4. Attempt any **four** parts of the following : **(3·5×4=14)**

(a) Prove that the set consisting of all the circuits and the edge-disjoint union of circuits (including the null set) in a graph G is an abelian group under the ring sum operation.

(b) Discuss the incidence matrix and adjacency matrix representation of a graph.

(c) Define :

(i) Cut-set matrix

(ii) Fundamental cut-set matrix.

Give example of each.

(d) Define the chromatic number of a graph. Prove that every tree with two or more vertices is 2-chromatic.

(e) Discuss the four color conjecture with suitable example.

(f) Characterize a graph for which the circuit space contains the vector (1, 1, ..., 1).