Printed Pages : 7 **TMA – 011 / MA - 011**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9972/9935 Roll No. | | | | | | | | | | |

B. TECH.

(SEM. VI) EXAMINATION, 2006-07

GRAPH THEORY

Time : 3 Hours] *[Total Marks : 100*

Note : *Attempt **all** questions.*

1 Attempt any **four** parts of the following : **5×4=20**

(a) Define a bipartite graph. Show that the complement of a bipartite graph need not to be a bipartite.

(b) Discuss the Konigsberg Bridge Problem.

(c) Define the following with one example each:
- (a) Infinite graph
- (b) Hamiltonian path
- (c) Component of a graph
- (d) Euler graph
- (e) Spanning subgraph

(d) Define isomorphic graph. Draw three isomorphic graph of the following graph.

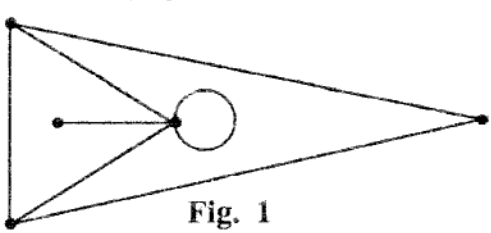


Fig. 1

uptuonline.com

(e) Differentiate, with example, a simple graph and a multigraph. Show that the maximum number of edges in a simple graph with n vertices n(n-1)/2.

(f) What is the largest number of vertices in a graph with

(a) 35 edges if all vertices are of degree at least 3.

(b) 24 edges and all vertices of the same degree

2 Attempt any **four** parts of the following: **5×4=20**

(a) Define binary tree and state two application of it in computer science.

(b) Apply Prime's algorithm to find a minimal spanning tree of the following graph.

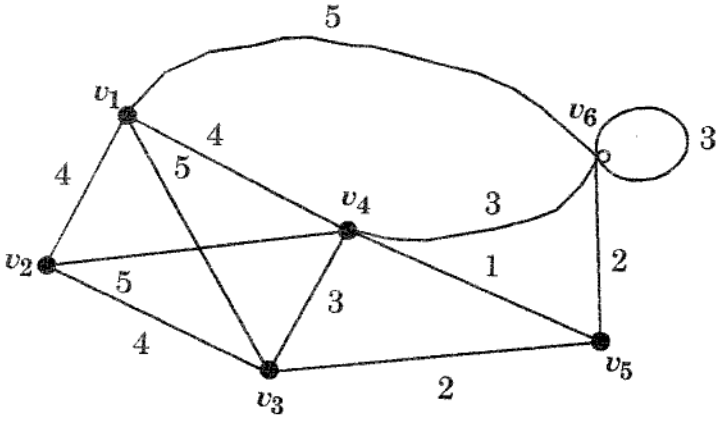


Fig. 2

(c) Find shortest path form v_i to v_8 using Dijkstra algorithm in the following graph.

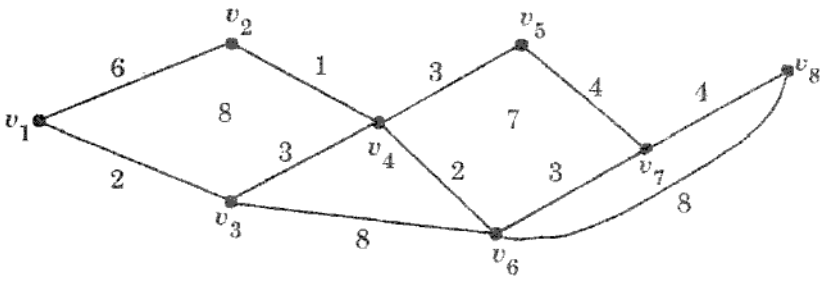


Fig. 3

(d) Define spanning tree of a graph. Show that a Hamiltonian path in a graph is a spanning tree.

(e) Show a tree in which its diameter in not equal to twice of the radius. Under what condition does this inequality hold? Elaborate.

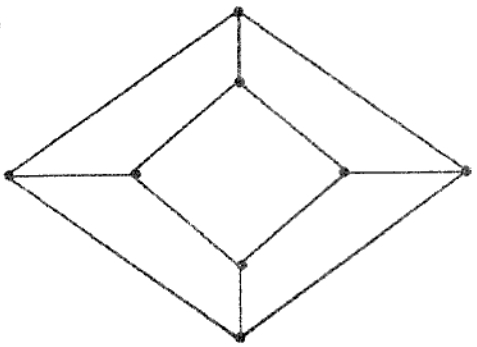
(f) What are the different properties when a graph G with n vertices is called a tree?

 uptuonline.com

3 Attempt any **four** parts of the following : **5×4=20**

(a) Define the edge connectivity and vertex connectivity of a connected graph. Find them for the following graphs :

(i)



(ii)

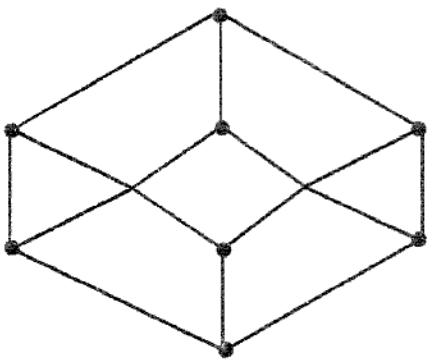


Fig. 4

uptuonline.com

(b) Show that a complete graph k_n is planar if $n \leq 4$.

(c) Draw a spanning tree of the following graph given below and list all the fundamental circuits with respect to this tree.

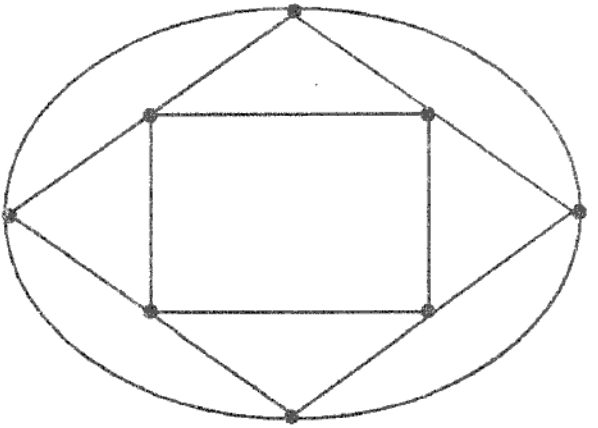


Fig. 5

(d) Find the dual of the following graph.

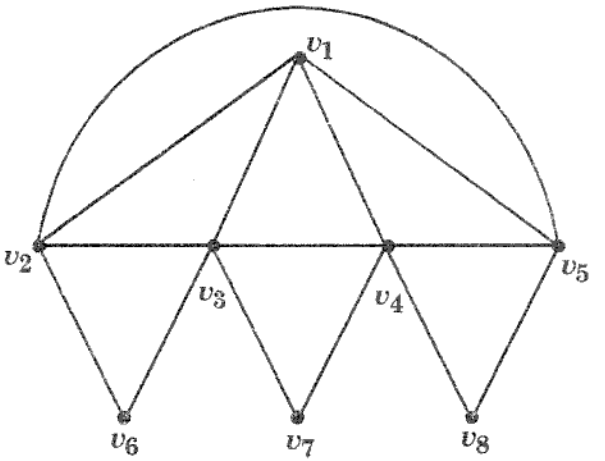


Fig. 6

uptuonline.com

(e) Prove that a graph G has a dual if and only if it is a planar.

(f) Show, by sketching, that the thickness of eight-vertex complete graph is two.

4 Attempt any **two** parts : **10×2=20**

(a) Define basis vectors of a graph. Find the number of distinct basis possible in a cut-set subspace.

(b) Define (i) reduced incidence matrix (ii) fundamental circuit matrix and (iiii) fundamental cut-set matrix, of a connected graph. Also derive the relationship between them.

(c) Consider the circuit matrix (B) and incidence matrix (A) of a simple connected graph whose columns are arranged using the same order of edges. Then prove that every row of B is orthogonal to every row of A. Also verify the result for the following graph.

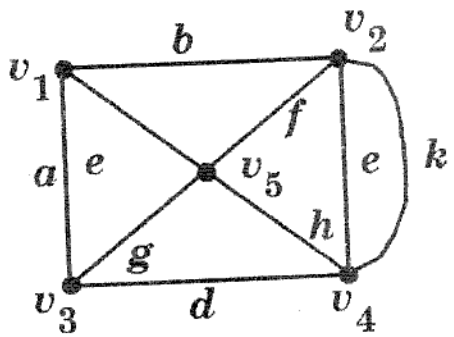
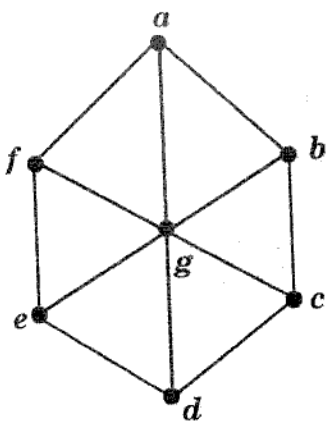


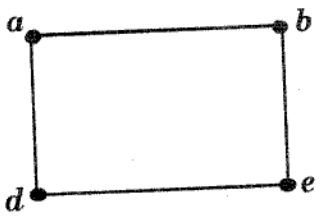
Fig. 7

5 Attempt any **two** parts : **10×2=20**

(a) What do you mean by chromatic number and chromatic polynomial of a graph? Determine the chromatic number and chromatic polynomial of the following graphs.



(i)



(ii)

Fig. 8

(b) Define Euler diagraph with example. Prove that every Euler diagraph without isolated verities is strongly connected. Also, show by constructing a counter example, that converse is not true.

(c) State and prove Cayley's theorem for counting trees.