Printed Pages - 4

TMA - 013

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID: 9974

| Roll No. |
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B.Tech.

(SEM VI) EVEN SEMESTER THEORY EXAMINATION, 2009-2010

PRINCIPLES OF OPERATION RESEARCH

Time: 3 Hours

Total Marks: 100

Note: Attempt **ALL** the questions.

1. Attempt any two parts of the following:

(2x10=20)

(a) Write simplex algorithm stepwise. Solve the following linear programming problem :

Maximize
$$x_1 + 5x_2 + 7x_3$$

Subject to
$$3x_1 + x_2 + 2x_3 \le 10$$

$$2x_1 + 5x_2 + x_3 \le 18$$

$$x_1 + 2x_2 + 7x_3 \le 28$$

and
$$x_1, x_2, x_3 \ge 0$$

(b) Solve using two - phase Method

Maximize
$$5x_1 + 4x_2 + 3x_3$$

Subject to
$$2x_1 + x_2 + 6x_3 = 200$$

$$6x_1 + 5x_2 + 10x_3 = 1000$$

$$8x_1 + 3x_2 + 6x_3 = 800$$

and
$$x_1, x_2, x_3 \ge 0$$

(c) Define the dual of a linear programming problem (LPP). Prove that the dual of the dual a LPP is the LPP itself.

2. Attempt any two parts of the following:

- (2x10=20)
- (a) Describe an algorithm to solve an integer LPP. Solve the following integer LPP :

Maximize
$$4x_1 + 2x_2 + 7x_3$$

Subject to
$$x_1 + 3x_2 + 5x_3 \le 15$$

$$2x_1 + x_2 + 3x_3 \le 10$$

and x_1 , x_2 , x_3 non-negative integers.

(b) Solve the following transportation problem with cost matrix as follows:

| | 1 | 22 | 3 | 4 | 5 | |
|---|-----|-----|-----|------|-----|------|
| A | 7 | 5 | 7 | 7 | 3 | 360 |
| В | 9 | 1 1 | 6 | 11 . | 5 | 250 |
| С | 11 | 10 | 6 | 6 | 8 | 350 |
| D | 9 | 15 | 9 | 2 | 12 | 40 |
| | 100 | 200 | 400 | 175 | 125 | 1600 |

(c) Formulate the following Capital Budgeting problem as a 0-1 integer linear programming problem given in the following data:

There are four projects under consideration. Assume that the projects run into three years. Total available funds are Rs. 75000, (to be used at the rate of Rs. 25000/- each year). The expected profit and cost break-up is as follows:

| Project | Expected Profit | Cost in Rs. | | | | |
|---------|-----------------|-------------|--------|--------|--|--|
| Tioject | Expected From | Year 1 | Year 2 | Year 3 | | |
| 1 | 90000 | 8000 | 10000 | 12000 | | |
| 2 | 60000 | 2000 | 5000 | 8000 | | |
| 3 | 1,80000 | 15000 | 10000 | 5000 | | |
| 4 | 1,00000 | 10000 | 5000 | 5000 | | |

3. Attempt any two parts of the following:

(2x10=20)

(a) The following table gives the activities of a construction project and duration:

| Activity | 1 - 2 | 1 - 3 | 2 - 3 | . 2 - 4 | 3 - 4 | 4 - 5 |
|--------------------|-------|-------|-------|---------|-------|-------|
| Duration (Days) | 20 | 25 | 10 | 12 | 6 | 10 |

- (i) Draw the network for the project
- (ii) Find the critical path
- (b) A small project is composed seven activities whose time estimates are listed in the table as follows:

| Activity | | Estimated duration (Week) | | | | | |
|----------|---|---------------------------|-------------|-------------|--|--|--|
| i | j | Optimistic | Most likely | Pessimistic | | | |
| 1 | 2 | - I | 1 | 7 | | | |
| 1 | 3 | 1 | 4 | 8 | | | |
| 1 | 4 | 2 | 2 | 1 | | | |
| 2 | 5 | 1 | 1 | 14 | | | |
| 3 | 5 | 2 | 5 | 7 | | | |
| 4 | 6 | 2 | 5 | 8 | | | |
| 5 | 6 | 3 | 6 | 15 | | | |

- (i) Draw the project network.
- (ii) Find the expected duration and variance of each activity.
- (iii) Calculate the early and late occurrence times for each event.
- (c) For a project consisting of several activities, the duration and the required resources for carrying out each of the activities and their availabilities are given below:

| Activity | Resourses Required | | | | | | |
|----------|--------------------|-----------|-----------------|--|--|--|--|
| Activity | Equipment | Operators | Duration (Days) | | | | |
| 1 - 2 | X | 30 | 4 , | | | | |
| 1 - 3 | Y | 20 | 3 | | | | |
| 1 - 4 | Z | 20 | 6 | | | | |
| 2 - 4 | X | 30 | 4 | | | | |
| 2 - 5 | Z | 20 | 8 | | | | |
| 3 - 4 | Y | 20 | 4 | | | | |
| 3 - 5 | Y | 20 | 4 | | | | |
| 4 - 5 | X | 30 | 6 | | | | |
| | | | | | | | |

- (i) Draw the network, identify critical path and compute the total float for each of the activities.
- (ii) Find the project completion time under given resource constraints. Resource availability: No. of operators 50

Equipment X = 1

Equipment Y = 1

Equipment Z = 1.

Attempt any two parts of the following:

(a) Define ECQ problem with no shortages and several production runs of unequal length.

An oil engine manufacturer purchases lubricants at the rate of Rs. 42 per piece from a vendor. The requirement of these lubricants is 1800 per year. What should be the order quantity per order, if the cost per placement of an order is Rs. 16 and inventory carrying charge per rupee per year is only 20 paise.

(b) A firm is considering replacement of a machine, whose cost price is Rs. 12200 and the scrap value Rs. 200. The running (maintenance and operating) costs in Rs. are found from experience to be as follows:

| Year | 79 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------|-----|-----|-----|------|------|------|------|------|
| Running Cost | 200 | 500 | 800 | 1200 | 1800 | 2500 | 3200 | 4000 |

When should the machine be replaced?

(c) The demand of an item is uniform at a rate of 20 units per month. The fixed cost is Rs. 10 each time a production run is made. The production cost is Re. 1 per item and inventory carrying cost is Rs. 0.25 per item per month. If the shortage cost is Rs. 1.25 per item per month, determine how often to make a production run and of what lot size it should be?

Attempt any two parts of the following:

 $(2\times10=20)$

(a) Divide a quantity b into n parts, so as to maximize their product.

Let f_n (b) denote the maximum value, show that :

$$f_1(b) = b$$
 and
 $f_n(b) = \text{Max} \{z, f_{n-1}(b-z)\}$

Hence find f_n (b) and the division that maximizes it.

(b) Using dynamic programming approach solve:

Max
$$z = x_1 x_2 x_3$$

 $x_1 + x_2 + x_3 \le 5$
 $x_1, x_2, x_3 \ge 0$

- (c) A company uses 10000 units per year of an item. The purchase price is Re. 1 per item. Ordering cost = Rs. 25 per order. Carrying cost per year is 12% of the inventory value.
 - (i) Find the EOQ.
 - (ii) Find the number of orders/year.
 - (iii) If the lead time is 4 week and assuming 50 working weeks per year, find the re-order point.