Printed Pages—3   **ECS072**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2709   Roll No. ☐☐☐☐☐☐☐☐☐☐

B.Tech.

(SEM. VII) THEORY EXAMINATION 2011-12
COMPUTATIONAL COMPLEXITY

Time : 3 Hours   *Total Marks : 100*

Note :—Attempt **all** questions.

1. Attempt any **two** parts of the following :   **(10×2=20)**

   (a) What are the different models of computation ? Describe these models in comparative manner.

   (b) Prove or disprove the following conjectures :

      (i) $f(n) = O(g(n))$ implies

      $g(n) = O(f(n))$

      (ii) $f(n) + g(n) = \theta(\min(f(n), g(n)))$

      (iii) $f(n) = O(g(n))$ implies

      $2^{f(n)} = O(2^{g(n)})$

   where $f(n)$ and $g(n)$ are asymptotically positive functions.

   (c) Are the following sets closed under the following operation. Prove your answer :

      (i) FIN (the set of finite languages) under the function odds L, defined on languages as follows :

      odds $(L) = \{w : \exists\, x \in L(w = odds(x)))\}$.

      (ii) INF(the set of infinite languages) under the function odds L.

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2. Attempt any **two** parts of the following :   **(10×2=20)**

   (a) What do you mean by complexity classes ? Discuss the relationship among the complexity classes.

   (b) Show that if there is a reduction from P_1 to P_2 then :

      (i) If P_1 is undecidable then so is P_2.

      (ii) If P_1 is non-recursive enumerable then so is P_2.

   (c) State and prove Rice theorem.

3. Attempt any **two** parts of the following :   **(10×2=20)**

   (a) Explain the general steps in establishing NP-completeness proof of a given problem.

   (b) Explain the Gödel's incompleteness theorem with the help of examples.

   (c) Write the randomized version of Quick sort algorithm.

4. Attempt any **two** parts of the following :   **(10×2=20)**

   (a) State the circuit satisfiability problem. Prove the circuit satisfiability problem belongs to the class NP.

   (b) Consider the problem of multiplication of an n×n matrix $A = (a_{ij})$ by an n-vector $x = (x_j)$. The resulting n-vector $y = (y_i)$ is given by the equation :

   $$y_i = \sum_{j=1}^{n} a_{ij} \ x_j \ ; \ \text{for i} = 1, 2, ..., n.$$

   Write an algorithm to perform matrix-vector multiplication by computing all the entries of y in parallel.

   (c) Write short note on Interactive proofs.

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5. Attempt any **two** parts of the following :   **(10×2=20)**

(a) Explain the completeness and soundness properties of probabilistically checkable proof system.

(b) Explain the following class of problems :

(i)   BPP

(ii) RP

(iii) CORP

(iv) ZPP.

(c) Write short note on Quantum computing.