

## B. TECH.

## SIXTH SEMESTER EXAMINATION, 2003-2004

## GRAPH THEORY

Time : 3 Hours

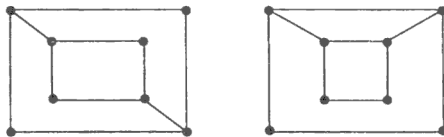
Total Marks : 100

Note : Attempt ALL questions.

1. Attempt any *FOUR* of the following :— (5×4=20)

(a) Prove that the sum of the degrees of the vertices of a graph is equal to twice the number of edges. Does the theorem hold for a multigraph ? Justify your answer with example.

(b) For the following pair of graphs, determine whether or not the graphs are isomorphic :—



Give the justification for your answer.

(c) Prove that a simple graph with  $n$  vertices and  $k$  components can have at most  $(n-k)(n-k+1)/2$  edges.

(d) Prove that a finite connected graph is Eulerian if and only if each vertex has even degree.

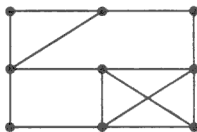
(e) Prove that, in a complete graph with  $n$  vertices, there are  $(n-1)/2$  edge disjoint Hamiltonian circuits, if  $n$  is odd number  $\geq 3$ .

(f) Define the following with one example each :—

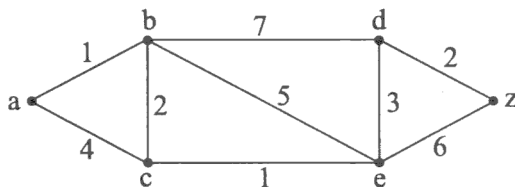
- (i) ☒ Subgraph
- (ii) ☒ Spanning subgraph
- (iii) ☒ Homeomorphic graphs
- (iv) ☒ Unicursal line
- (v) ☒ Arbitrarily traceable graphs

2. Attempt any *FOUR* of the following :— (5×4=20)

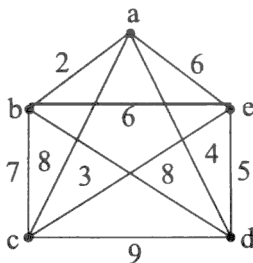
- (a) If  $G$  is tree with  $n$  vertices then prove that it has exactly  $n-1$  edges.
- (b) Explain what is meant by a spanning tree. Find four spanning trees for the following graph :—



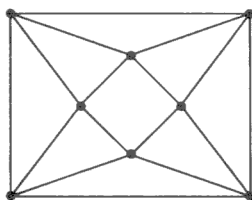
- (c) Find the shortest path from  $a$  to  $z$  of the following graph using Dijkstra Algorithm :—



- (d) Use the algorithm of Kruskal to find a minimum weight spanning tree in the following graph :—

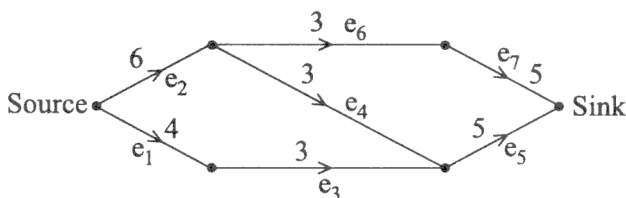


- Take any spanning tree in the following graph. List all the seven fundamental cut-sets with respect to this tree :—



3. Attempt any *FOUR* of the following :— (5×4=20)

- Draw a graph with  
Edge connectivity = 4  
Vertex connectivity = 3  
Degree of every vertex  $\geq 5$
- Show that the complete bipartite graph  $K_{3,3}$  is non-planar.
- In a simple connected planar graph  $G$ , there are  $r$  regions,  $v$  vertices ( $v \geq 3$ ) and  $e$  edges ( $e > 1$ ) then
  - $e \geq 3r/2$
  - $e \leq 3V - 6$
  - there is a vertex  $V$  of  $G$  s.t. degree ( $V$ )  $\leq 5$ .
- Prove that a graph has a dual if and only if it is planar.
- Show, by sketching, that the thickness of nine-vertex complete graph is three.
- Use Ford and Fulkerson algorithm to find the maximum flow of the network :—



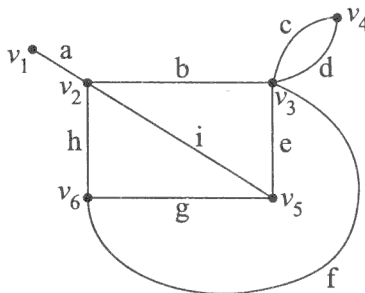
$$\frac{Q(G)}{e}$$

$$\frac{36}{e}$$

$$37.9 - 6$$

$$\frac{e}{21}$$

- (a) What is the difference between incidence and adjacency matrices ? Prepare both matrices for given graph :—



- (b) Define the terms with example :—
- (i) Circuit matrix
  - (ii) Cut-set matrix
  - (iii) Fundamental Cut-set matrix

Also prove that the rank of cut-set matrix is equal to the rank of graph and rank of incidence matrix.

- (c) Explain the dot product of two vectors and orthogonal vectors. Prove that the dot product of two vectors, one representing a subgraph  $g$  and other the  $g'$ , is zero if the number of common edges to  $g$  and  $g'$  is even and the dot product is 1, if the number of common edges is odd.

5. Attempt any TWO of the following :—, (10×2=20)

- (a) Prove that  $m$ -vertex graph is a tree iff its chromatic polynomial is  $P_m(n) = n(n-1)^{m-1}$ .

- (b) Define Arborescence with example. Discuss its one application. Also prove that an arborescence is a tree in which every vertex other than root has an in-degree of exactly one.
- (c) What do you understand by enumeration of graphs ? Explain it. Discuss types of enumeration. Also prove that the number of simple labelled graphs of  $n$  vertices is  $2^{n(n-1)/2}$ .