Printed Pages—4 **MA—012**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9936 **Roll No.** | | | | | | | | | | |

002003

B.Tech.

SIXTH SEMESTER EXAMINATION, 2005-2006

PROBABILITY AND STOCHASTIC PROCESS

Time : 3 Hours *Total Marks : 100*

Note : *(i)* *Answer **ALL** questions.*

(ii) *All questions carry equal marks.*

(iii) *In case of numerical problems assume data wherever not provided.*

(iv) *Be precise in your answer.*

1. Attempt *any four* parts of the following : **(5x4=20)**

(a) If A and B are two events , prove that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

What happens when the events are independent ?

(b) A card is drawn from a well-shuffled pack of playing cards. What is the probability that it is either a spade or an ace ?

(c) Let p(r) denotes the probability of r successes in binomial distribution so that $P(r) = n_{Cr}\, p^r q^{n-r}$

If the $\lim_{\substack{n\to\infty \\ p\to 0}} np = \lambda$, then prove that

$$p(r) = \frac{\lambda^r e^{-\lambda}}{r!}.$$

Name this distribution.

(d) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a poisson distrubution with mean 1.5. Calculate the proportion of days on which car is not used and the proportion of days on which some demand is refused.

(e) Show that for the geometric distrubution $q^x p$, $x=0,1,2,........,\ q=1-p$,

(i) mean is less than variance,

(ii) recurrence formula is $p(x+1) = qp(x)$.

(f) Define the hypergeometric distribution and find its mean.

2. Attempt *any four* parts of the following : **(5x4=20)**

(a) If the probability density function of continuous random variable x is

$f(x) = kx^{\alpha-1}(1-x)^{\beta-1}$, for $0<x<1,\ \alpha>0,\ \beta>0$

$= 0$ elsewhere,

Find k and mean of x.

(b) Show that in the normal distribution,

$$\mu_{2n} = 1.3.5.....(2n-5)(2n-3)(2n-1)\sigma^{2n},$$

Where μ_{2n} is the even order moments about the mean.

(c) The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months ?

Given : $P(t \geq 2) = 0.0228$

where $t = \dfrac{x-\mu}{\sigma}$, σ is the mean and μ is the s.d .

(d) A general form of negative exponential distribution with parameter λ (>0) is given by

$$f(x)=\begin{cases} 0 & \text{for } x\le 0 \\ \lambda e^{-\lambda x} & \text{for } x>0 \end{cases}$$

Find the moment generating function of f(x) about x=0.

(e) If X is uniformly distributed over [1,2], find z such that $P(X>Z+\mu_x)$.

(f) A line of given length a is divided into three parts. What is the probability that these form the sides of a triangle ?

3. Attempt *any two* parts of the following : **(10x2=20)**

(a) (i) Find the mean and variance of the following probability distribution given by

x :	4	8	12	16	20
f(x):	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{12}$

(ii) An umbrella merchant can earn Rs. 30 per day if it rains on that day. If it is fair he loses Rs.6 per day. What is his expectation per day if the probability of rain on any day is 0.3 ?

(b) The probabilities of three independent events are p_1, p_2 and p_3. Find the probability that

(i) exactly one ,

(ii) none,

(iii) at least one , of these three will happen ?

(c) Suppose that two independently functioning components, each with the same constant failure rate λ are connected in parallel. Find E(T) of the system.
[Hint : Here E(T) means the mean of the distribution].

4. Attempt *any two* parts of the following : **(10x2=20)**

(a) Describe Bernoulli Process in brief. If T_r represents the r^{th} order interarrival time, then prove that

$$E(T_r) = \frac{r}{p} \text{ and } \operatorname{Var}(T_r) = \frac{r(1-p)}{p^2}.$$

(b) If M(t) denotes the removal function, show that

$$M(t) = F(t) + \sum_{n=1}^{\infty} F^{(n+1)}(t), \text{ where } \overset{(n)}{F(t)} \text{ is the } n^{th} \text{ fold}$$

convolution of F with itself.

(c) Write down the Pallaezek- Khinchin mean -value formula. State if there is relation between this formula with M|D|1. Explain M|M|1.

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) Under the M|M|m queue, if λ and μ are the birth - death rate,
Prove that

$$P_k = P_0 \left(\frac{\lambda}{\mu}\right)^K \frac{1}{m!\, m^{k-m}}, \quad k \geq m.$$

(b) Discuss the machine repairman model.

(c) Show that the variance of the number of persons in the queue is $\frac{\rho}{(1-\rho)^2}$ where $\rho = \frac{\lambda}{\mu}$.

- o O o -