$\qquad$ Roll No: $\square$

## B. TECH. <br> (SEM-III) THEORY EXAMINATION 2019-20 DISCRETE STRUCTURES

Time: 3 Hours
Total Marks: 100
Note: Attempt all Sections. If require any missing data; then choose suitably.
SECTION - A

1. You are required to answer all the parts of this question.
$2 \times 10=20$
(a) Define power set.
(b) Find $A \triangle B$ if $A=\{1,2,5\}, B=\{3,4,5,8\}$
(c) What is order of an element in a Group?
(d) Define Ring.
(e) In an integral domain $D$, show that if $x y=x z$ with $x \neq 0$ then $y=z$.
(f) In any Boolean algebra, show that $(a+b) \cdot\left(a^{\prime}+c\right)=a^{\prime} \cdot b+a c+b c$
(g) Define quantifiers.
(h) Draw truthtable of proposition $p \vee \sim p$.
(i) Explain conditional proposition.
(j) What is Hamiltonian cycle?

## SECTION - B

2. Attempt any three parts of the following:
(a) What are ordered pair and Cartesian product? Prove that

$$
A \times(B \cap C)=(A \times B) \cap(A \times C)
$$

(b) Explain Groupoid and Monoid with necessary conditions and example.
(c) How are sequential circuits different from combinational circuits? Draw the logic circuit corresponding to Boolean expression $\mathrm{Y}=\mathrm{A}+\mathrm{BC}+\mathrm{B}$.
(d) Prove that the following propositions are tautology
i. $\sim(p \wedge q) \vee q$
ii. $p \rightarrow(p \vee q)$
(e) Solve the following recurrence relation:
i. $a_{n}+a_{n-1}=3 n 2^{n}$
ii. $a_{n}=2 a_{n-1}+3^{n}, n \geq 1$ and $a_{0}=1$ (by iterative method)

> SECTION - C
3. Attempt any one part of the following:
(a) What is recursion? Explain any recursively defined function with suitable example.
(b) What is closure of relation? Explain Symmetric and transitive closure in detail.
4. Attempt any one part of the following:
(a) Prove that inverse of each element in a group is unique.
(b) Let $G=(1,-1, i,-i\}$ with the binary multiplication be an algebraic structure, where $i^{2}=-1$,
(i) Determine whether G is an Abelian
(ii) If G is cyclic group, then determine the generator of G
5. Attempt any one part of the following:
(a) Draw hasse diagram for divisibility relation on following set $A=$ $\{3,4,12,24,48,72\}$
(b) Simplify the following Boolean functions using three variable maps $\mathrm{F}(\mathrm{x}, \mathrm{y}, \mathrm{z})=\sum(0,1,5,7)$
$\mathrm{F}(\mathrm{x}, \mathrm{y}, \mathrm{z})=\sum(1,2,3,6,7)$
6. Attempt any one part of the following:
(a) Prove by mathematical Induction
i. $\quad 6^{n+2}+7^{2 n+2}$ is divisible by 43 for every positive integer $n$.
ii. $\quad n!\geq 2^{n-1}$ for $n \geq 1$
(b) What is normal form? Find principal disjunctive normal form of
i. $\quad p \rightarrow q$
ii. $\quad q \vee(p \vee \sim q)$
7. Attempt any one part of the following:
(a) Determine the GF of a numeric function $a_{r}$ where $a_{r}=\left\{\begin{array}{l}2^{r} \text { if } r \text { is even } \\ -2^{r} \text { if } r \text { is odd }\end{array}\right.$
b) Explain the following:
i. Directed Graph
ii. Weighted Graph
iii. Bipartite Graph
iv. Null Graph
v. Complete Graph

