Printed Pages : 4 **TEE-301**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2047 Roll No.

B. Tech.

(SEM. III) EXAMINATION, 2007-08

BASIC SYSTEM ANALYSIS

Time : 3 Hours] *[Total Marks : 100*

Note : *Attempt all questions.*

1 Attempt any **four** parts of the following : **5×4=20**

(a) Calculate energies and powers of the following signals : **5**

(i) $e^{-5t}u(t)$ (ii) $2\sin 2t + 3\cos \pi t$

(b) Calculate the following integrals : **5**

(i) $\int_{-\infty}^{5} (5+\cos t)\,\delta(t-10)\,dt$

(ii) $\int_{-\infty}^{\infty} (t-2)^2\,\delta(t-2)\,dt$

(iii) $\int_{-\infty}^{\infty} e^{-at^2}\,\delta(t-10)\,dt$

(c) Differentiate between time-invariant and time-variant systems. Give a suitable example for both types of systems. **5**

(d) What do you understand by linear systems ? Show that the system described by the following differential equation is linear **5**

$$\dot{y}(t) + t\,y(t) = r(t)$$

(e) Sketch the waveforms of the following signals : **5**

(i) $x(t) = u(t) - u(t-2)$

(ii) $x(t) = u(t+1) - 2u(t) + u(t-1)$

(iii) $y(t) = r(t+1) - r(t) + r(t-2)$

where $u(t)$ and $r(t)$ are unit step and ramp signals respectively.

(f) Consider the R-L-C series circuit in **Fig. 1** which is closed at $t = 0$. Write the loop equation for this circuit assuming zero initial condition. What are three different possible situations of the transient solution of above equations ? Describe the corresponding relationship R, L and C for each of the situations. **5**

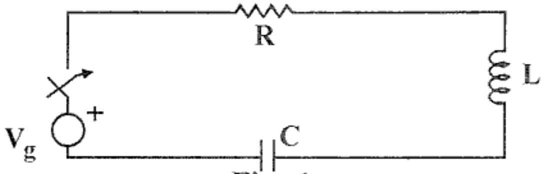

Fig. 1

2 Attempt any **two** parts of the following : **2×10=20**

(a) Determine the effect of each of the following symmetry conditions on the coefficients of the Fourier series expansion for $f(\theta)$ and obtain the formula for those coefficients which do not vanish **10**

(1) $f(\theta) = f(\pi - \theta)$ (2) $f(\theta) = -f(\pi - \theta)$

(b) What are Dirichlet's conditions ? **10**

(1) Find the Fourier transform of $x(t) = \delta(t)$

(2) Find the inverse Fourier transform of $X(jw) = 2\pi\,\delta(w)$.

(c) Find the Fourier series of the signal shown in **Fig. 2.** using exponential form. **10**

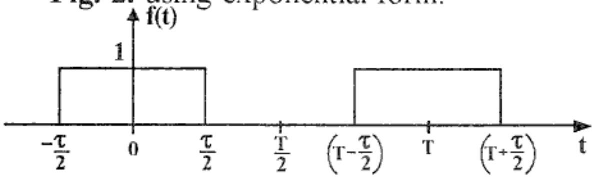

Fig. 2

3 Attempt any **two** parts of the following : **2×10=20**

(a) Use the Laplace transform to determine the output of a system represented by the differential equation **10**

$$\ddot{y}(t) + 5\,\dot{y}(t) + 6\,y(t) = 2\,\dot{x}(t) + x(t)$$

in response to the input $x(t) = u(t)$. Assume that all initial conditions are zero.

(b) Prove the following results : **10**

(1) $L\left[y\left(\dfrac{t}{a}\right)\right] = aY(s),\ a > 0$

(2) $L[ty(t)] = \dfrac{d}{ds}Y(s)$

where $Y(s)$ is Laplace transform of $y(t)$. **10**

(c) Find the Laplace transform of the wave form shown in **Fig 3**. It is to be noted that $v(t) = 0$ for $t > 2T$ and $t < 0$.

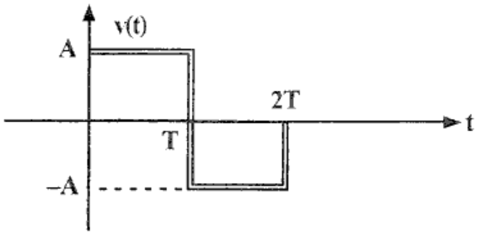

Fig. 3

4 Attempt any **two** parts of the following : **2×10=20**

(a) Find the state transition matrix for **10**

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

Find the corresponding resolvent matrix also.

(b) Find the output response of the system described by the following state variable formulation. **10**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$ to unit step input

It is given that $C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $x^T(0) = [1 \;\; 1]$.

(c) Consider the state variable model of a second-order system represented as **10**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} r$$

$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $r =$ unit step.

Find the state response $x(t), \; t > 0$

5 Attempt any **two** parts of the following : **2×10=20**

(a) Find the Z–transform of the sequences **10**

(i) $\delta(k)$ (ii) $u(k)$ (iii) $e^{\pm\beta k}$; $k \geq 0$

(b) Prove that $\lim\limits_{z \to \infty} F(z) = f(0)$ **10**

$$\lim_{k \to \infty} f(k) = \lim_{z \to 1} \left[\left(\frac{z-1}{z} \right) F(z) \right]$$

where $F(z)$ is Z transform of $f(k)$.

(c) Obtain the Z-inverse of $F(z)$ for the following : **10**

(1) $\dfrac{z}{(z-0.4)}$, $|z| > 0.4$

(2) $\dfrac{Z}{(z-0.4)}$, $|z| < 0.4$